Putting Vagueness in Context

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University of Auckland,
Philosophy Department Seminar

1 Tolerance & Indeterminacy

Tolerance	$(\mathsf{Dark(n)}) \to \mathsf{Dark(n-1)}$
NSC	$\neg(\mathsf{Dark(n)} \wedge \neg\mathsf{Dark(n-1)})$
Modus Ponens	$\phi, \phi \to \psi \models \psi$
Non-Contradiction	$\neg(\phi \wedge \neg\phi)$
Bivalence	$\mathsf{True}(\phi) \vee \mathsf{False}(\phi)$
T-Schema	$\phi \leftrightarrow \mathsf{True}(\phi)$
Polarity	$\mathsf{True}(\neg\phi) \leftrightarrow \mathsf{False}(\phi)$
LEM	$\phi \vee \neg\phi$

2 Language

- $\mathsf{Dark(n)} \in \mathcal{L}$, if $1 \leq n \leq 30$.
- $\neg\phi$, $\mathsf{True}(\phi)$, $\mathsf{False}(\phi) \in \mathcal{L}$ if $\phi \in \mathcal{L}$.
- $\phi \wedge \psi$, $\phi \to \psi \in \mathcal{L}$, if ϕ, $\psi \in \mathcal{L}$.

3 Contexts

A context, $c = \langle c^+, c^- \rangle$, is a pair of subsets of $\mathcal{D}$ satisfying (i) non-triviality, (ii) convexity and (iii) partiality. $c \preccurlyeq c'$ iff $c^+ \subseteq c'^+$ and $c^- \subseteq c'^-$.

- c **incorporates** ϕ iff $c[\phi]^+c$.
- c **excludes** ϕ iff $c[\phi]^-c$.
- $c \models \phi$ iff there is no c' such that $c \preccurlyeq c'$ and c' excludes ϕ.
- $\phi_1, ..., \phi_n \models \psi$ iff for all c, if $c[\phi_1]^+, ...[\phi]^+c'$, then $c' \models \psi$.

4 Semantics

i. $[\mathsf{Dark(n)}]^+ = \{\langle c, c' \rangle \mid c \preccurlyeq c' \wedge n \in c'^+ \wedge \neg\exists c'' : c \preccurlyeq c'' \prec c' \wedge n \in c''^+\}$

$[\mathsf{Dark(n)}]^- = \{\langle c, c' \rangle \mid c \preccurlyeq c' \wedge n \in c'^- \wedge \neg\exists c'' : c \preccurlyeq c'' \prec c' \wedge n \in c''^-\}$

ii. $[\neg\phi]^+ = [\phi]^-$

$[\neg\phi]^- = [\phi]^+$

iii. $[\phi \wedge \psi]^+ = \{\langle c, c' \rangle \mid c[\phi]^+ \circ [\psi]^+ c'\}$

$[\phi \wedge \psi]^- = \{\langle c, c' \rangle \mid c[\phi]^- c' \vee c[\psi]^- c'\}$

iv. $[\phi \to \psi]^+ = \{\langle c, c \rangle \mid \forall c' : c[\phi]^+ c' \supset c' \models \psi\}$

$[\phi \to \psi]^- = \{\langle c, c \rangle \mid \exists c' : c[\phi]^+ c' \wedge c' \not\models \psi\}$

v. $[\mathsf{True}(\phi)]^+ = \{\langle c, c \rangle \mid \langle c, c \rangle \in [\phi]^+\}$

$[\mathsf{True}(\phi)]^- = \{\langle c, c \rangle \mid \langle c, c \rangle \notin [\phi]^+\}$

vi. $[\mathsf{False}(\phi)]^+ = \{\langle c, c \rangle \mid \langle c, c \rangle \in [\phi]^-\}$

$[\mathsf{False}(\phi)]^- = \{\langle c, c \rangle \mid \langle c, c \rangle \notin [\phi]^-\}$